0.1 Using the Limit Definition of a Derivative**

1. The function f gives the voltage (in Volts) across a capacitor as a function of time t (in seconds) since the capacitor started being charged. Write the meaning for each of the following expressions in the context of this situation.

   (a) $f(5) - f(2)$

   (b) $\dfrac{f(t+4) - f(t)}{4}$

   (c) $\lim\limits_{\Delta t \to 0} \dfrac{f(t+\Delta t) - f(t)}{\Delta t}$

2. Consider the function defined by $f(x) = x^2 - 1$. What is the value of $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{(3+h) - 3}$?

   (a) 0

   (b) 6

   (c) 8

   (d) $2x$

   (e) The limit does not exist.

3. The expression $\lim\limits_{h \to 0} \dfrac{(x+h)^3 - \ln(x+h) - (x^3 - \ln(x))}{h}$ is the derivative of what function?

   (a) $f(x) = (x+h)^3 - \ln(x+h)$

   (b) $f(x) = 3x^2 - \dfrac{1}{x}$

   (c) $f(x) = 3x^2 - \dfrac{1}{x}$

   (d) $f(x) = x^3 - \ln(x)$

   (e) $f(x) = \dfrac{(x+h)^3 - \ln(x+h) - (x^3 - \ln(x))}{h}$

4. Evaluate $\lim\limits_{h \to 0} \dfrac{5\left(-1+h\right)^9 - 5\left(-1\right)^9}{h}$.

   (a) 45

   (b) -45

   (c) 5

   (d) -5

   (e) Limit does not exist

5. If f is a differentiable function and a is a number, then $f'(a)$ is given by which of the following expressions:

   I. $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

   II. $\lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$

   III. $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{x-h}$

   a. I only
   b. II only
   c. I and II only
   d. I and III only
   e. I, II, and III

6. The following expression represents the derivative of what function?

$$\lim_{\Delta x\to 0} \frac{3\cos^2(x+\Delta x)-3\cos^2(x)}{\Delta x}$$

   a. $f(x)=3\cos^2(x+\Delta x)$
   b. $f(x)=3\cos^2(x)$
   c. $f(x)=3\cos^2(x+\Delta x)-3\cos^2(x)$
   d. $f(x)=6\cos(x)\sin(x)$
   e. $f(x)=\dfrac{3\cos^2(x+\Delta x)-3\cos^2(x)}{\Delta x}$

7. $\lim_{h\to 0} \dfrac{(2+h)^4-2^4}{h} =$

   a. 0
   b. 16
   c. 1
   d. 32
   e. The limit does not exist